46| Претходно саопштење
Preliminary report
doi 10.7251/STP1813632L ISSN 2566-4484

KONSOLIDACIJA POMOĆU VERTIKALNIH DRENOVA

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Rezime:
Izgradnja nasipa saobraćajne infrastrukture (puteva i železnica) zahteva što bržu i kvalitetniju gradnju. Jedan od problema koji se najčešće javlja u ostvarivanju ovih ciljeva je konsolidacija koja se često odvija još dugo nakon završetka izgradnje, a usled čega nastaju naknadna sleganja i oštećenja saobraćajnica. Zbog toga se primenjuju mere da se konsolidacija ubrza i završi za vreme građenja nasipa.
U okviru ovog rada se prikazuje jedna od metoda ubrzanja konsolidacije pomoću vertikalnih drenova. Pored teorijskih razmatranja konsolidacije daje se i jedan primer primene ove metoda.

Ključne reči: infrastruktura, nasip, konsolidacija, sleganje, vertikalni drenovi

CONSOLIDATION WITH VERTICAL DRAINS

Abstract:
Building the embankment of traffic infrastructure (roads and railways) requires faster and more quality construction. One of the problems that often occurs in achieving these goals is the consolidation, which often occurs long after the completion of construction, resulting in subsequent settlements and damage to the roads. Therefore, certain measures are being implemented to accelerate and complete consolidation during embankment construction.
In this paper, one of the methods of accelerating consolidation, using vertical drains, is shown. In addition to theoretical considerations of consolidation, one example of the application of this method is given.

Keywords: infrastructure, embankment, consolidation, settlement, vertical drains

1. INTRODUCTION

In cohesive materials, especially if their pores filled with water, subsidence are not present but may develop months or even years. In the paper on seepage and seepage effects it was assumed that the volume occupied by the water per unit of volume of the soil is independent of the state of stress in the soil. There is no real soil which strictly satisfies the continuity condition, because every change in the state of stress produces a certain change in the volume of voids per unit volume of the soil. Yet, if the soil is very permeable and not very compressible, the change of the porosity due to a change in the state of stress in the soil can usually be disregarded.

Bearing in mind all this, the ground where time consolidation is a problem for the dynamics of the construction of, in particular embankments, apply measures to accelerate consolidation and finishes during the construction of the embankment.

2. THEORY OF CONSOLIDATION

The application of total stress (or a load) to an unsaturated soil can result in the generation of excess pore-air and pore-water pressures. The excess pore-air and pore-water pressures will dissipate with time and eventually the pore pressures return to the equilibrium values that existed prior to loading. The dissipation process of pore pressures is called “consolidation” and the process results in volume change as excess pore pressures are dissipated. It is also possible for excess pore fluid pressures to be generated as a result of changes in boundary conditions. For example, the infiltration of rainwater at ground surface can initiate water movement into the soil along with associated soil swelling or soil collapse. Water moves into the soil because of a difference in the hydraulic head (or pore-water pressure) at ground surface and the hydraulic head immediately within the soil mass. The drying of a soil from the ground surface also initiates moisture movement that can be accompanied by volume decrease. The drying of the soil can be visualized in terms of a moisture flux at ground surface or a change in the pore-water pressure boundary condition.

The “swelling” process is usually associated with a change in moisture flux boundary conditions whereas the consolidation process is usually associated with a change in the externally applied total stresses. In the case of a swelling process, the pore pressures (i.e., pore-water pressure, pore-air pressure or both) are below the equilibrium state and therefore will increase toward an equilibrium state. The consolidation

process has a decrease in pore pressures with time while the swelling process has an increase in pore pressures with time. The consolidation and swelling processes are essentially equivalent and opposite processes from a theoretical standpoint. However, the swelling process is not usually initiated through a change in the total stresses (i.e., unloading of the soil). Rather, it is usually environmental changes related to precipitation that directly change the pore-water pressure conditions at the ground surface and initiate a process of pore-water pressure changes throughout the soil mass.

If the load on a layer of saturated soil such as clay isincreased, the layer is compressed, and excess water drains out of it. This constitutes a process of consolidation. During the process the quantity of water thatleaves a thin horizontal slice of the soil is larger than thequantity that enters it. The difference is equal to thedecrease in volume of the layer; thus the continuity condition expressed by Eq.1 (since the volume of water in the element is n dx dy dz, where n is the porosity, the equation of continuity) [1,2]:

$$\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)dx\,dy\,dz=-\frac{\partial}{\partial t}\left(n\,dx\,dy\,dz\right) \qquad (1)$$

The added pressure or load per unit of area that produces the consolidation is known as the consolidation pressure increment. At the instant of its application, it is carried almost entirely by the water in the voids of the soil. Therefore, at the beginning of a process of consolidation, there is an initial excess pressure in the water almost exactly equal to the consolidation pressure increment. As time goes on, the excess pore-water pressure decreases, and the effective vertical pressure in the layer correspondingly increases. At any point within the consolidating layer, the value *u'* of the excess pore-water pressure at a given time may be determined from

$$u' = u - u_s \qquad (2)$$

in which u is the total pore-water pressure and u_s is the reference static or steady-state pore-water pressure in the consolidating layer. At the end of primary consolidation the excess pore-water pressure u' becomes equal to zero, and the entire consolidation pressure increment becomes an effective stress transmitted through the structure of the soil.

The Darcy equation in terms of excess pore-water pressure is:

$$v_z=-\frac{k_v}{\gamma_w}\frac{\partial u'}{\partial z} \qquad (3)$$

Assuming that the coefficient of permeability k_v is the same at every point in the consolidating layer and forevery stage of consolidation, and expressing the porosity n in terms of void ratio e, we obtain

$$\frac{k_v}{\gamma_w}\frac{\partial^2 u'}{\partial z^2}=\frac{1}{1+e}\frac{\partial e}{\partial t} \qquad (4)$$

Equation (4) is the hydrodynamic equation of onedimensional consolidation based on the assumptions that the coefficient of permeability is constant and the strains are small during consolidation.

Even in the absence of time-dependent changes in total stress and groundwater level or reference pore-water pressure, the compression of saturated soils is time-dependent because it is the result of two separate mechanisms, each of which is time-dependent. This can be illustrated by the following basic equation relating void ratio, effective stress, and time

$$\frac{de}{dt}=\left(\frac{\partial e}{\partial \sigma_v'}\right)\frac{d\sigma_v'}{dt}+\left(\frac{\partial e}{\partial t}\right)_{\sigma_v'}=a_{vs}\frac{d\sigma_v'}{dt}+a_{vt} \qquad (5)$$

where the subscripts *v, s* and *t* denote vertical, stress, and time, respectively.

If we assume that the time lag of the compression is caused exclusively by the finite permeability of the soil,so that in Eq. 5 a_{vt} = 0, and if we assume further that in Eq. 5 a_{vs}

is equal to - a_v , which is the same at every point in the layer and for every stage of consolidation, then Eq. 5 becomes

$$\frac{de}{dt} = a_v \frac{d\sigma_v'}{dt} \qquad (6)$$

If the total vertical stress σ_v, and the reference pore-water pressure u_s remain unchanged during consolidation, then $d\sigma_v'/dt=-du'/dt$ and Eq. 4 becomes

$$\frac{k_v}{\gamma_w}\frac{\partial^2 u'}{\partial z^2} = \frac{a_v}{1+e}\frac{\partial u'}{\partial t} \qquad (7)$$

In terms of $m_v=a_v/(1+e)$, where $m_v=\Delta\varepsilon_v/\Delta\sigma_v'$ and ε_v is vertical strain, we have

$$\frac{1}{\gamma_w}\frac{k_v}{m_v}\frac{\partial^2 u'}{\partial z^2} = \frac{\partial u'}{\partial t} \qquad (8)$$

By introducing the coefficient of consolidation c_v defined as

$$c_v = \frac{1}{\gamma_w}\frac{k_v}{m_v} \qquad (9)$$

we obtain

$$c_v \frac{\partial^2 u'}{\partial z^2} = \frac{\partial u'}{\partial t} \qquad (10)$$

The dependent variable *u'* is a function of the independent variables *z* and *t*. In the partial differential Eq. 10, *u'* is differentiated twice with respect to *z* and once with respect to *t*. Consequently, the solution of Eq. 10 requires two boundary conditions in terms of *z* and an initial condition in terms of *t*.

The differential Eq. 10 can be solved subject to any set of initial and boundary conditions to obtain an expression for the excess pore-water pressure. A solution using the Fourier expansion method leads to

$$u'(z,t) = \Delta\sigma_v \sum_{m=0} \frac{2}{M}\sin\left(M\frac{z}{H}\right)\exp(-M^2 T_v) \qquad (11)$$

where $M=\pi(2m+1)/2$

$$T_v = \frac{c_v t}{H^2} \qquad (12)$$

is a pure number called the *time factor*, and *H* is the maximum drainage distance [5].

The degree of compression of a sublayer during consolidation is

$$U(z,t) = \frac{e_0 - e}{e_0 - e_p} \qquad (13)$$

where e_p is the void ratio when the excess pore-water pressure becomes zero. Because a linear time-independent relationship between void ratio and effective vertical stress is assumed in formulating the theory of consolidation represented by Eq. 10, the degree of compression defined by Eq. 13 is identical with the degree of effective vertical stress increase. This in turn is equal to the degree of excess pore-water pressure dissipation

$$U(z,t) = \frac{u_i' - u'}{u_i'} \qquad (14)$$

The degree of consolidation of the layer of thickness H is

$$U = \frac{s}{s_p} \qquad (15)$$

where s_p is the settlement of the layer when the excess pore-water pressure becomes zero throughout the thickness H. The expression for U as a function of the time factor T_v, is obtained by integrating with respect to z the degree of excess pore-water pressure dissipation of the sublayers. For example, from Eq. 11 we obtain

$$U = 1 - \sum_{m=0} \frac{2}{M^2} \exp(-M^2 T_v) \qquad (16)$$

Several $U - T_v$ curves are shown in Fig. 1. For an open layer (thickness $2H$) the relationship between U and T_v, is determined by Eq. 16 and the curve C_1 for all cases in which the initial excess pore-water pressure varies linearly with z.
If the consolidation pressure increment for a half-closed layer decreases from some value $\Delta\sigma_{vt}$ at the top to zero at the bottom, the relation between U and T_v is given by the curve C_2. If it increases from zero at the top to $\Delta\sigma_{vb}$ at the bottom. Figure 1.b shows the curves C_1, to C_3 plotted to a semilogarithmic scale from which small values of U can be obtained more accurately. In the arithmetic plot, Fig. 1.a**,** the initial part of the curve C_1 has a parabolic shape. In fact, up to a degree of consolidation of 60% the relation between U and T_v, is accurately defined by $U = 2\sqrt{T_v / \pi}$.

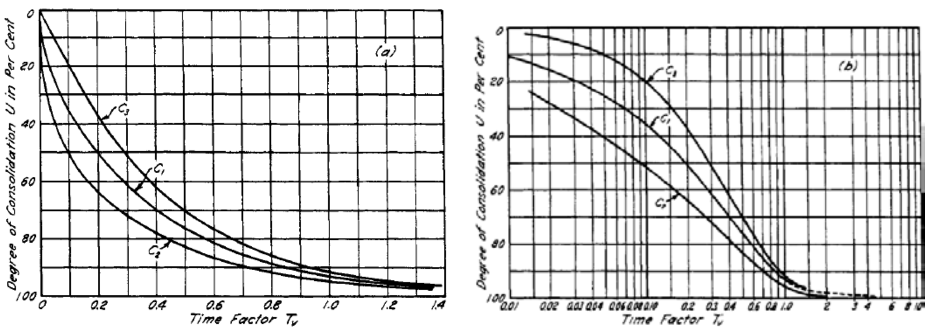

Figure 1. Relation between degree of consolidation and time factor. In (a) the time factor is plotted to an arithmetic and in (b) to a logarithmic scale [2]

3. VERTICAL DRAINS

In theory the final magnitude of consolidation settlement is the same, only the rate of settlement being affected [3,4].

In the case of an embankment constructed over a highly compressible clay layer, Fig. 2, vertical drains installed in the clay would enable the embankment to be brought into service much sooner and there would be a quicker increase in the shear strength of the clay. A degree of consolidation of the order of 80% would be desirable at the end of construction. Any advantages, of course, must be set against the additional cost of the installation.
The traditional method of installing vertical drains is by driving boreholes through the clay layer and backfilling with a suitably graded sand. Typical diameters are 200–400mm and drains have been installed to depths of over 30 m. The sand must be capable of allowing the efficient flow of water while preventing fine soil particles from being washed in. Careful backfilling is essential to avoid discontinuities which could give rise to 'necking' and render a drain ineffective. Necking could also be caused by lateral soil displacement during consolidation.

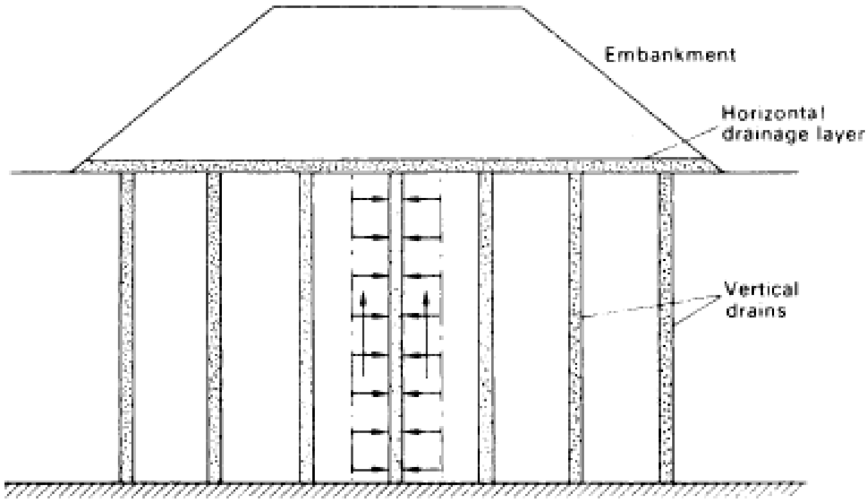

Figure 2. Vertical drain [3]

Vertical drains may not be effective in overconsolidated clays if the vertical stress after consolidation remains less than the preconsolidation pressure. Indeed, disturbance of overconsolidated clay during drain installation might even result in increased final consolidation settlement. It should be realized that the rate of secondary compression cannot be controlled by vertical drains.
In polar coordinates the three-dimensional form of the consolidation equation, with different soil properties in the horizontal and vertical directions, is

$$\frac{\partial u_c}{\partial t} = c_h\left(\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial u_c}{\partial r}\right) + c_v\frac{\partial^2 u_c}{\partial z^2} \qquad (17)$$

The vertical prismatic blocks of soil surrounding the drains are replaced by cylindrical blocks, of radius R, having the same cross-sectional area (Figure 3).
The solution to Equation 17 can be written in two parts: $U_v=f(T_v)$ and $U_r=f(T_r)$ where U_v = average degree of consolidation due to vertical drainage only; U_r = average degree of consolidation due to horizontal (radial) drainage only.
Time factor for consolidation due to radial drainage only defined Equation 12.

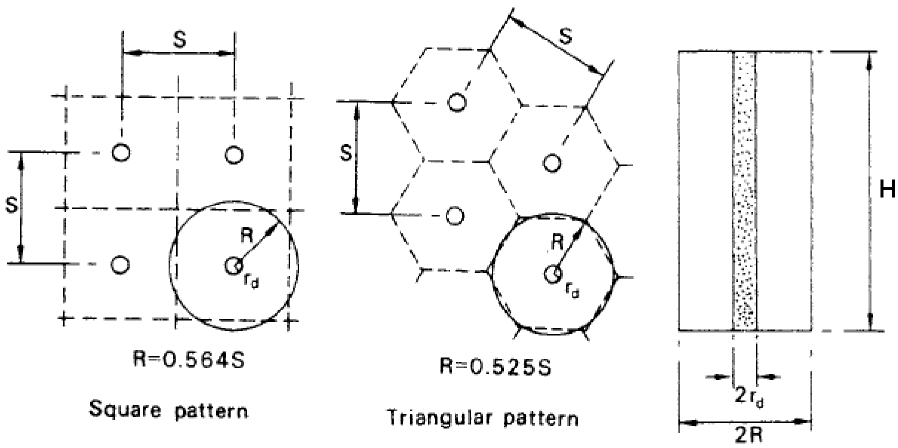

Figure 3. Cylindrical blocks [3]

The expression for T_r confirms the fact that the closer the spacing of the drains, the quicker the consolidation process due to radial drainage proceeds. The solution for radial drainage, due to Barron, is given in Figure 4, the U_r /T_r relationship depending on the ratio $n = R/r_d$, where R is the radius of the equivalent cylindrical block and r_d the radius of the drain. It can also be shown that

$$(1\text{-}U) = (1\text{-}U_v)(1\text{-}U_r) \qquad (18)$$

where U is the average degree of consolidation under combined vertical and radial drainage.

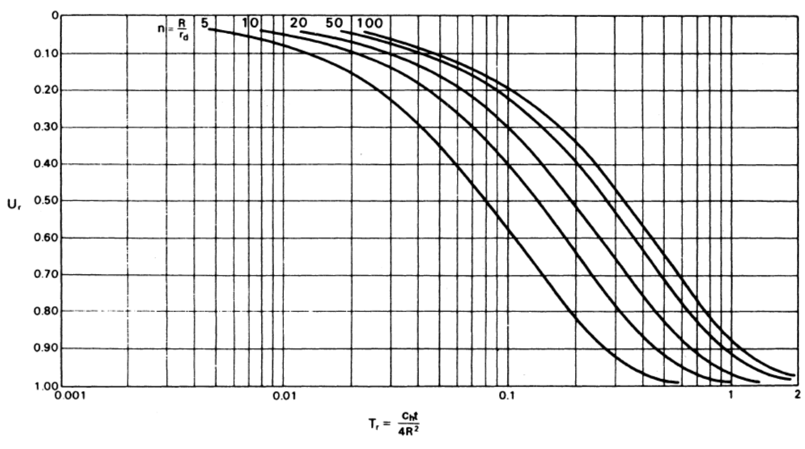

Figure 4. Solution for radial consolidation [3]

In large-scale soil, consolidation shuffling is rapidly performed, immediately after loading, because resistance to water withdrawal from the breach is very small. In fine-grained soils, consolidation depends to a large extent on the speed of water evacuation from the pores. When the layer of clay or soil is low in high-water impermeability, the acceleration of consolidation by pre-loading alone will not be effective, as a large period of time is required to achieve significant shuffling. In this case, vertical drains are installed, which reduce the drainage path, and therefore the time of consolidation. Vertical drains have a direct impact on the acceleration of primary consolidation, but have no direct impact on secondary consolidation. The earlier completion of the primary consolidation results in the earlier beginning of the secondary consolidation. This is also the greatest importance of the drains. It should also be emphasized that the acceleration of consolidation would be successful, it would be necessary to allow the pressurized layer to extrude water from the layer.

4. NUMERICAL EXAMPLE

For the purposes of the construction of the plateau, geotechnical tests were carried out on the basis of which the budget was prepared, as stated in this paper. Numerical example is the real.

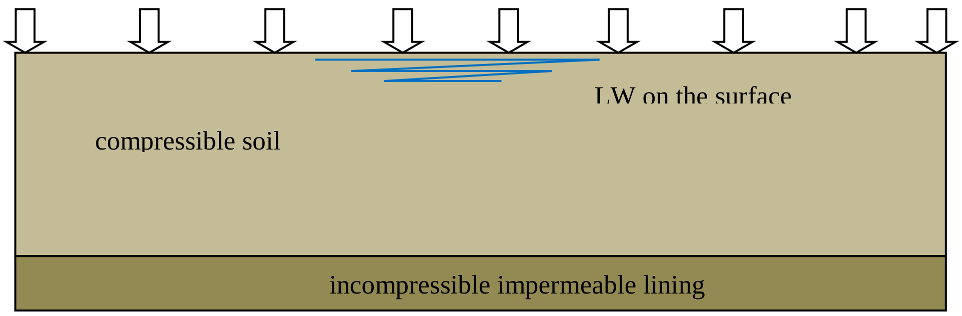

Figure 5. Cross-section of the terrain below the embankment

Characteristic compressible layer

- layer thickness $H = 5.40$ m
- saturated volume weight $\gamma_z = 18.50$ kN/m^3
- submerged volume weight $\gamma' = 18.50\text{-}9.81 = 8.69$ kN/m^3
- coefficient of vertical consolidation $c_v = 2.00*10^{-3}$cm^2/sec $= 2.315*10^{-2}$ m^2/dan
- coefficient of horizontal consolidation $c_h = 3*c_v = 6.945*10^{-2}$ m^2/dan
- compressibility module $M_v = 3000$ kN/m^2

Load of an embankment – q

The average height of the embankment $H_n = 4.00$ m

$q_n = 20.00$ kN/m^3*4.00 m = 80.00 kN/m^2

Consolidation settlement

Budget through compression modulus

$S_c = H*q/M_v^{100\text{-}200} = 540*80.00/3000 = 14.40$ cm

70 cm above the middle of the placenta

Calculation of the consolidation time for vertical drainage in the natural state
Consolidation Time:
$t = (T_v * H^2)/c_v = (T_v * 4.00^2)/ 2.315*10^{-2} = 690*T_v$
Diagram Figure 1a [2] - Characteristic curve for unilateral drainage - C1

Table 1.Calculation primary consolidation

U%	10	20	30	40	50	60	70	80	90
T_v	0.008	0.031	0.071	0.126	0.197	0.289	0.403	0.567	0.846
t(дани)	6	21	49	87	136	199	278	391	584

Time required for primary consolidation = 584 days = 19.50 months = 1.6 years.
Additional measures to accelerate consolidation - installation of wick drains.

Calculation of consolidation time for radial drainage with vertical drains (wick drains)
Characteristics of wick drains

- width lines b = 100 mm
- thickness lines d = 3 mm
- equivalent drain diameter $d_w = 2*(b+d)/pi= 2*(100+3)/3.14 = 65.6$ mm
- diameter of the disturbed zone $d_s = 4*d_w = 4*65.6 = 262.4$ mm => s = 4
- the drains are installed in a triangular raster with a distance of 1.00 m
- an equivalent diameter of action $D_e = 1.05*1.00$ m = 1050 mm
- n = 1050/65.6 = 16
- consolidation coefficient horizontal $c_h = 6.945*10^{-2}$ m^2/dan

Consolidation Time:
$t = T_r*D_e^2/c_h = 1.05^2*T_r / 6.945*10^{-2} = 15.9\ T_r$

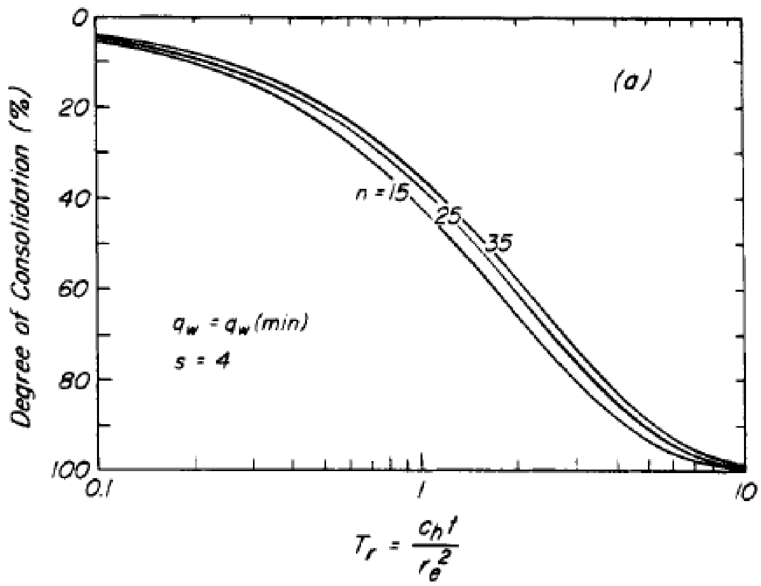

Figure 5. Diagram time- degree of consolidation [2]

Table 2. Calculation primary consolidation – wick drains

U_r%	10	20	30	40	50	60	70	80	90
T_r	0.205	0.428	0.664	0.955	1.282	1.702	2.242	3.005	4.306
t(дани)	3	7	11	15	20	27	37	48	69

The time required for primary consolidation t = 69 days is shorter than the estimated time of embankment construction. In the construction of plateau is significantly shortened the time of consolidation.

5. CONCLUSIONS

Modern design and construction of roads implies an ever-wider application of geotechnical measures in the construction of embankments. Investors, especially roads, require construction works to be carried out as soon as possible. The construction of embankments, especially from local materials, is in collision with this requirement. Therefore, the issue of acceleration of consolidation is getting more and more important. This condition makes geotechnical research as detailed as possible, and in mind also methods of accelerating consolidation, among other things, vertical drains.
The paper is presenting theoretical considerations of consolidation and one of the methods of accelerating consolidation, using vertical drains, is shown.

Acknowledgement: The paper is result of the investigation in the projects OI 174027 and TP 36043 financed by the Ministry of Science and Technological Research of Republic of Serbia

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